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vertices of the triangle are then

$$\begin{aligned} x_{23} &= A_1/C_1, & y_{23} &= B_1/C_1. \\ x_{31} &= A_2/C_2, & y_{31} &= B_2/C_2. \\ x_{12} &= A_3/C_3, & y_{12} &= B_3/C_3. \end{aligned} \tag{16}$$

The coördinates of the circumcenter are seen by (11) and (16) to be[1]

$$\begin{aligned} C_x &= \frac{1}{2D^2}\left| \frac{A_i^2+B_i^2}{C_i}, \quad B_i, \quad C_i \right|, \\ C_y &= \frac{1}{2D^2}\left| A_i, \quad \frac{A_i^2+B_i^2}{C_i}, \quad C_i \right|. \end{aligned} \tag{17}$$

The coördinates of the centroid are

$$C_x = \tfrac{1}{3}\left(\frac{A_1}{C_1}+\frac{A_2}{C_2}+\frac{A_3}{C_3}\right), \qquad C_y = \tfrac{1}{3}\left(\frac{B_1}{C_1}+\frac{B_2}{C_2}+\frac{B_3}{C_3}\right). \tag{18}$$

The coördinates of the orthocenter are

$$\begin{aligned} O_x &= -\frac{1}{C_1C_2C_3D^2}\begin{vmatrix} (A_2A_3+B_2B_3)C_1^2, & B_1, & C_1 \\ (A_3A_1+B_3B_1)C_2^2, & B_2, & C_2 \\ (A_1A_2+B_1B_2)C_3^2, & B_3, & C_3 \end{vmatrix}, \\ O_y &= -\frac{1}{C_1C_2C_3D^2}\begin{vmatrix} A_1, & (A_2A_3+B_2B_3)C_1^2, & C_1 \\ A_2, & (A_3A_1+B_3B_1)C_2^2, & C_2 \\ A_3, & (A_1A_2+B_1B_2)C_3^2, & C_3 \end{vmatrix}. \end{aligned} \tag{19}$$

The area of the triangle is $D^2/(2C_1C_2C_3)$.

A few of the above results are well known; the others are easily proved. This paper is intended only as an introduction to work along this line and is not supposed to be exhaustive in any sense.

III. The History of Mathematics in Elementary Instruction.[2]

By Laurin Zilliacus, Bedales School, Petersfield, England.

The General Education Board has recently announced the appropriation of $16,000 to be used by the National Committee on Mathematical Requirements in financing a study looking to improvements in the mathematical curriculum of the secondary schools of America. If there is to be any recasting of curricula, I earnestly hope that a full use will be made of the history of mathematics. A short perusal of this history, as given in popular works such as Ball's *History of Mathematics*, would, I feel sure, cause a majority of present-day teachers of mathematics to modify considerably both the outlines of their courses and their methods of presenting the subject. The particular lines of change which would result I am not qualified to predict, but I think it would be safe to say in general that (1) there would be a reduction in unnecessary drudgery as a true perspective of values was obtained; (2) the fundamental principles (*e.g.*, compact number notation, symbolic statement, variation, limits, deductive reasoning) would stand out more clearly and comprehensively when it was seen how they had been gradually pulled out from confusion, instead of their being, as at present, presented as part of a complete and arbitrary structure; (3) there would be a living connection between mathematics and "practical" life in the minds of children who were shown how each has waited on the other throughout the ages.

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[1] Since $|A_1B_2C_3| = |a_1b_2c_3|^2$.

[2] Extract from a letter to Professor H. W. Tyler.

My own experience in a school of boys and girls of 11 to 19 years of age has convinced me that the coördination of mathematics with experimental science and the historical treatment of mathematics go hand in hand and together yield most fruitful results. A couple of lectures in the school on the history of mathematics roused great interest and led to many questions and some independent reading. A chart was displayed of the "Stream of Mathematical Knowledge" from the earliest known times, showing how the thin trickle of isolated facts from the Egyptians was swelled by the Greeks into a great river of knowledge; how the river branched and wandered, increasing here and shrinking there, through the Arabs (with a great tributary from the Hindoos), the Romans, the Byzantines and the Moors, until with the advent of the printing press and the flight of the Byzantine Greeks from Constantinople the streams reunited in Europe, and swelled on every hand—there the chart ended, but it is to be continued later. I found that the children were delighted as with a story, and that mathematics acquired a new meaning for them when linked up with historical familiars such as Athens, Alexandria, the Mohammedan conquests, Haroun al Raschid, the Moors of Spain, etc., etc.

In particular classes, too, the history was of great service. My difficulties with a Mechanics class vanished when a discovery in books (Cox's *Mechanics*) enabled me to outline the development of our views of the solar system from the theory of epicycles through the Copernican system, Galileo's trouble-bringing discoveries and Kepler's Laws to Newton's great work. The class was deeply interested, and grasped Newton's Laws, circular motion, energy and momentum in a way I have never before experienced with a class.

A beginner's class in geometry was much helped by a short account of how Thales and his successors took experimentally determined facts from the Egyptians, investigated their explanation and deduced other useful facts from them: the meaning of a proof was quickly grasped, and very rapid progress made in what they regarded as an intellectual game. They themselves discovered experimentally the property of angles in a semicircle, and were probably as eager as Thales to find the explanation and proof.

Summing up the benefits that struck me even during my short experience of teaching mathematics with a due regard for its history, I should say they are as follows:

For the teacher: (1) The relative importance of various aspects of the subject becomes easier to determine, and hence what should be omitted and what emphasized.

(2) Indications are obtained of the order in which to introduce new knowledge, since the history of the world's progress in mathematics is not without similarity to that of an individual (*e.g.*, the late appearance in history of an understanding of the minus sign in algebra indicates where it should be attempted in the school study of this subject).

(3) The difficulties of beginners are more readily seen, for they are much the same throughout time. (A striking instance of this is the confusion between determinate and indeterminate problems—once pointed out, it never recurs.)

For the taught: (4) An understanding of mathematics as a body of methods and knowledge that have grown and are still growing by the labours of men, and not as a curiously pointless and arbitrary set of rules and definitions.

(5) A better grasp of underlying principles as the course of history makes them stand out more and more clearly.

(6) A connection of mathematics with interesting and useful inventions.

(7) The interest that any subject gains that is packed with anecdotes and stories.

IV. AN INTRODUCTION TO PLANE TRIGONOMETRY BY GRAPHICAL METHODS.

By H. J. ETTLINGER, University of Texas.

It is not an uncommon experience of teachers of trigonometry to find students, who are plunged without warning into the definitions of the trigonometric functions, completely bewildered by the terms, sine, cosine, etc. It may be a good plan in taking a cold bath to immerse suddenly, but it is equally a good plan in teaching trigonometry to lead the student into the subject by a method which will directly connect it with his previous study of plane geometry.

This summer in a course in trigonometry the writer developed at some length a graphical introduction, based on the use of the ruler, compasses, and protractor. This graphical substratum continued throughout the course as an essential element.

As a student, the writer recalls a frequent remark of one of his instructors, Professor Roever of Washington University, that the element of value contained in an accurately drawn figure is usually completely overlooked. The habit of drawing by means of instruments careful and accurate figures to represent mathematical situations should be developed as early as possible. Often such a figure suggests a solution; always it will provide a check. It serves admirably as an example of what modern mathematical rigor means, viz: clearness.

The writer puts forward no claim to originality in this paper. A number of the ideas are to be found in the early chapters of Wilczynski and Slaught's *Plane Trigonometry* (Allyn and Bacon). In the course as given the past summer,